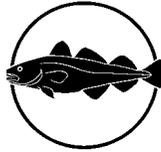# The Faroese Fisheries Laboratory

Fiskirannsóknarstovan

A Mathematical Model of the Primary Production on the Faroe Shelf with Cylinder Bottom Topography

by

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Technical Report No.: 04-03

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1 Introduction

In this technical report the marine ecosystem model of the lowest trophical levels on the Faroe Shelf is reviewed with respect to horizontal variations. Other documentation on the model can be found in Technical Reports 02-01, 03-01 and 04-02 (Eliasen *et al*, 2002, Eliasen *et al*, 2003 and Eliasen, 2004).

2 Model with a cylinder bottom topography

In December 2003 a new method was approached in the marine ecosystem model for the Faroe Shelf. The part of the model concerning growth of the organisms is described in Technical Report No. 04-02, and here we implement horizontal variations in the model.

A cylinder bottom topography based on the real bottom topography is generated, by sorting grid points depending on the depths in the bottom topography matrix, and from this, the area of each annulus is computed. The sorting of data is based on a matlab routine from Larsen, (Larsen, pers. comm.), and the properties of the cylinder bottom topography can be seen in Table 6-1 (section 6), and Figure 2-1.
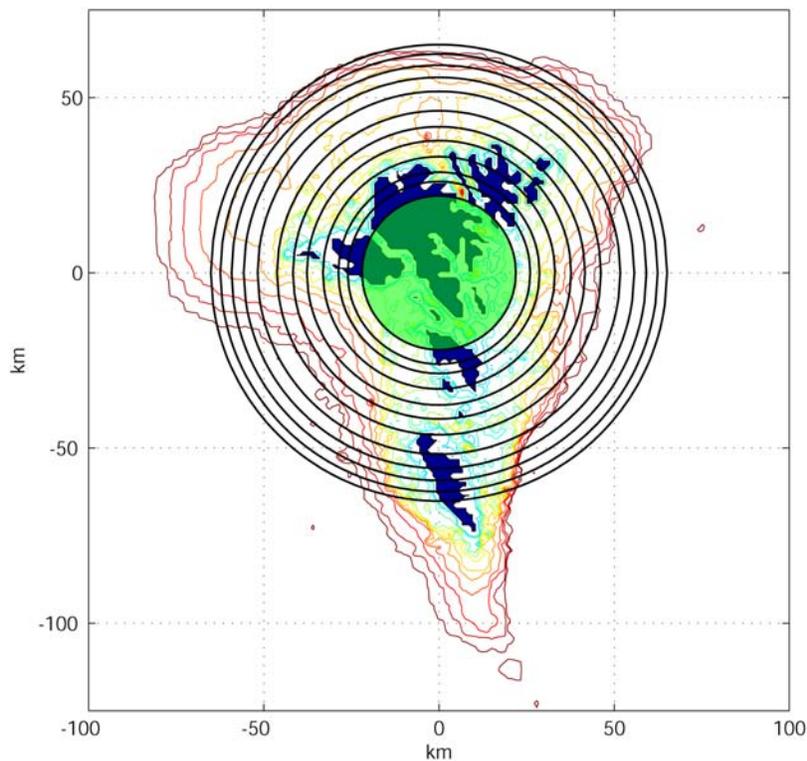

Figure 2-1. Bottom topography in the model. The green centre represents the concentrated land in the model, while black circles are iso-baths, with the first line outside land being 50m's depth, and then the circles are plotted with 10m's interval out to 150m's depth. Underneath, the real topography is plotted in colours with the same contour intervals as the black contours.
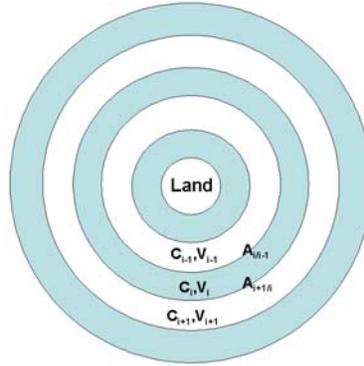
3   Horizontal exchange of water

Figure 3-1. Schematic drawing of the model.

Between subsequent cylinders there is exchange of water and of course other particles. The size of the exchange is proportional to the difference in concentration multiplied with a horizontal exchange coefficient k, estimated in other work by Larsen (2003).

The exchange between rings is computed as:

$$\frac{dC_i}{dt} = \frac{kA_{i+1/i}(C_{i+1} - C_i) - kA_{i/i-1}(C_i - C_{i-1})}{V_i}$$

Equation 3-1

where $i+1$, i and $i-1$ refer to subsequent rings (see Figure 3-1), C is concentration, V is volume, A is the exchange area between two rings and k is the horizontal exchange coefficient.

3.1   The horizontal exchange coefficient

In the cylinder bottom model there is a limitation on how large a horizontal exchange coefficient can be applied in the model.

Referring to Figure 3-1, the dimensions of the horizontal exchange coefficient has an upper limit, determined by the size of the rings and the timestep Δt. Assuming that the concentration in ring $i-1$ and $i+1$ is zero, we must demand that after one timestep, the concentration in ring $i-1$ or $i+1$ must not be higher than the concentration in ring i. The difference in concentration is computed in Equation 3-1. When assuming that all the concentrations are zero except the concentration in ring i, the differential equations are given as:

$$\frac{dC_{i-1}}{dt} = \frac{kA_{i/i-1}C_i}{V_{i-1}}; \qquad \frac{dCi}{dt} = \frac{-kA_{i+1/i}C_1 - kA_{i/i-1}C_i}{V_i}; \qquad \frac{dC_{i+1}}{dt} = \frac{kA_{i+1/i}C_i}{V_{i+1}};$$

Then after one timestep we have:

$$C_{i-1} + \frac{dC_{i-1}}{dt}\Delta t \leq C_i + \frac{dC_i}{dt}\Delta t \quad \text{and} \quad C_{i+1} + \frac{dC_{i+1}}{dt}\Delta t \leq C_i + \frac{dC_i}{dt}\Delta t .$$

Inserting the known expressions for the differential equations, we get the condition for k:

$$k \leq \frac{1}{\Delta t \left[\frac{A_{i/i-1}}{V_{i-1}} + \frac{A_{i+1/i}+A_{i/i-1}}{V_i} \right]}$$

Equation 3-2

Depending on the resolution of the bottom model and the timestep we here have a condition for the maximum size of the horizontal exchange coefficient. In this model we use a resolution of the bottom model with intervals of 10 m depth, with the innermost ring being out to 50m depth. With a timestep of half an hour we get a maximum k=43km/d, which certainly is sufficiently high. Other estimates have shown that a typical value for the horizontal exchange coefficient is k_{typ}=1.45km/d (Larsen, 2003), which is almost 30 times lower than the maximum horizontal exchange coefficient.

We have an exchange of water between off- and on-shelf water. The exchange time as a function of the horizontal exchange coefficient is seen in Figure 3-2. It is computed as the time it takes to renew the volume on shelf:

$$T = \frac{V}{A \cdot k}$$

Equation 3-3

With the horizontal exchange coefficient mentioned earlier (k_{typ}=1.45km/d), the exchange time is 13.5 days, see Figure 3-2, black curve. From other work it is estimated that the exchange time is around 2½ months (Gaard and Hansen 2000), which corresponds to an exchange rate of k=0.26km/d.
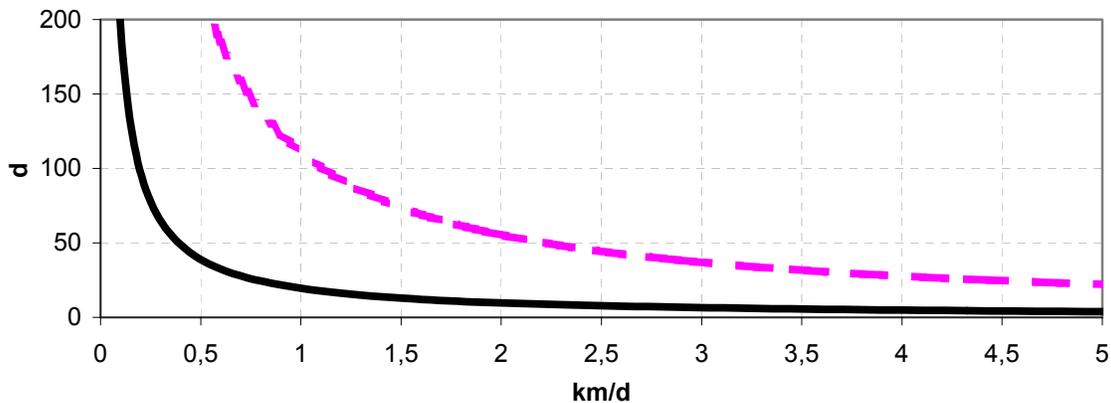

Figure 3-2   Exchange time as a function of horizontal exchange coefficient. Black solid curve is computed by Equation 3-3. Pink dashed curve is the time it takes to increase the concentration in the innermost ring from 0 to 50% of the off shelf concentration, when initially the concentration on the shelf is 0.

This method of evaluating the exchange time has a limitation. In reality the water in the innermost area is not renewed as Equation 3-3 indicates because it only estimates the volume exchanged through the shelf - off shelf boundary. If we define the exchange time as the time it takes to increase the concentration in the innermost ring from 0 to 50% of the off shelf concentration, when we initially demand the concentration on shelf to be 0 and off shelf 1, we get a more realistic result, see Figure 3-2, pink dashed curve and

section 4.1. With this assumption the average exchange time is 76 days, which corresponds well with other estimates.

3.2 Boundary conditions

In this model the same components as before are computed, i.e. detritus, nutrients, diatoms, flagellates, *C. finmarchichus* and benthos (Eliasen, 2003). Of course benthos does not move with the water masses, and is omitted in the horizontal exchange equations. On shelf, the changes in concentration are computed in each time step, while the values off shelf are boundary values and have to be predetermined. Off shelf all parameters are zero except nutrients and *C. finmarchichus*, which have functions describing their behaviour as a function of time in the top 150 m, see Figure 3-3 and Figure 3-4. The equations describing the curves shown in these figures are given as:

$$
C.finm._U = \begin{cases}
7\,\tfrac{ind}{m^3} \cdot 0.75 & 0 < JD \le 50 \\[2mm]
(-0.347 \cdot JD^2 + 7.8223 \cdot JD - 297.4)\tfrac{ind}{m^3} \cdot 0.75 & 50 < JD \le 120 \\[2mm]
(-0.2924 \cdot JD^2 + 108.98 \cdot JD - 8757.1)\tfrac{ind}{m^3} \cdot 0.75 & 121 < JD \le 155 \\[2mm]
7\,\tfrac{ind}{m^3} \cdot 0.75 & 155 < JD \le 365
\end{cases}
$$

Equation 3-4

$$
N_U = \begin{cases}
12\,\tfrac{\mu molN}{kg} & 0 < JD \le 134 \\[2mm]
(0.0008 \cdot JD^2 - 0.3532 \cdot JD + 44.931)\tfrac{\mu molN}{kg} & 134 < JD \le 307 \\[2mm]
12\,\tfrac{\mu molN}{kg} & 307 < JD \le 365
\end{cases}
$$

Equation 3-5

where JD is the Julian day number.

The nutrients are computed in μmolN/kg and the *C. finmarchicus* in ind/m^3. To convert the *C. finmarchicus* concentration to mgC/m^3, the weight of C6 is multiplied with 0.14 mgC/ind (Heath *et al.*, 2000). Equation 3-4 is deduced from estimates of numbers of *C. finmarchicus* in the topmost 50m observed on R/V Magnus Heinason, with the assumption that the total concentration in the topmost 150m is 75% of the concentration in the top 50m. This is perhaps too high, but at the moment we do not want to underestimate the *C. finmarchicus* influence in the ecosystem.

It is assumed that *C. finmarchichus* are advected onto the shelf from the outside, and therefore, initially there is no *C. finmarchichus* on shelf. The input parameters can be seen in Table 6-2 and the equations used in section 6.1. The results from this modelling can be seen in section 4.2.
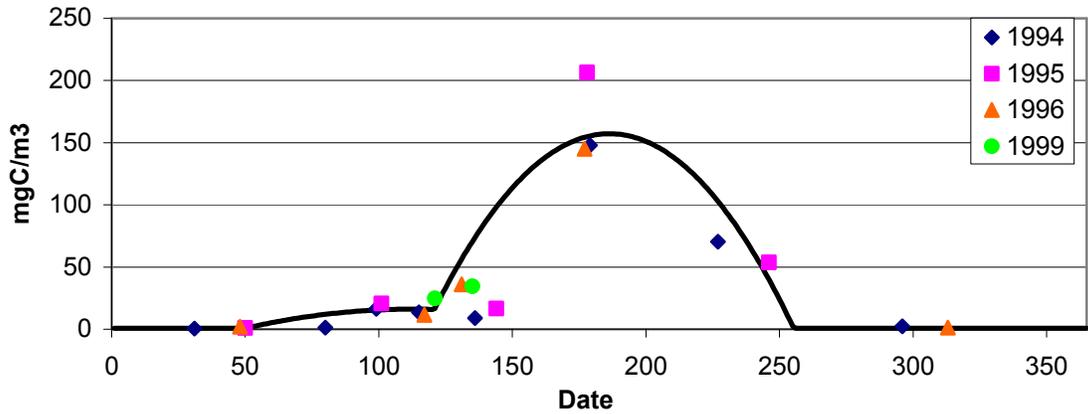
Figure 3-3. Black curve: Maximum concentration of C4-C6 *C. finmarchicus* off shelf in top 150 m during the year. The curve is estimated as a maximum of off shelf observations in top 50 m (coloured dots). The observations are from 1994-1996 and 1999 and are obtained on R/V Magnus Heinason cruises. It is assumed that the concentration in the top 150m is 75% of the concentration in the top 50m.
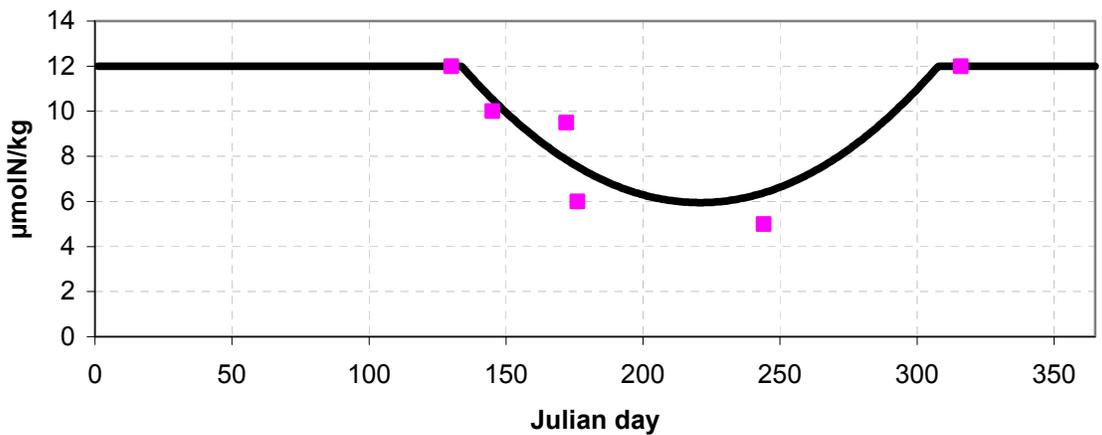
Figure 3-4. Nitrate concentration off shelf in topmost 150 m during the year. Based on observations from 1995 (Gaard, 1996), pink dots.

Although we usually observe very few *C. finmarchicus* on shelf pre bloom, some years we see the same concentration of *C. finmarchicus* on shelf as off shelf as soon as *C. finmarchicus* is assumed to ascend from the deep water. One year is 1996, where we already in the start of April observe similar concentrations of *C. finmarchicus* on shelf as off shelf, see Figure 3-5.

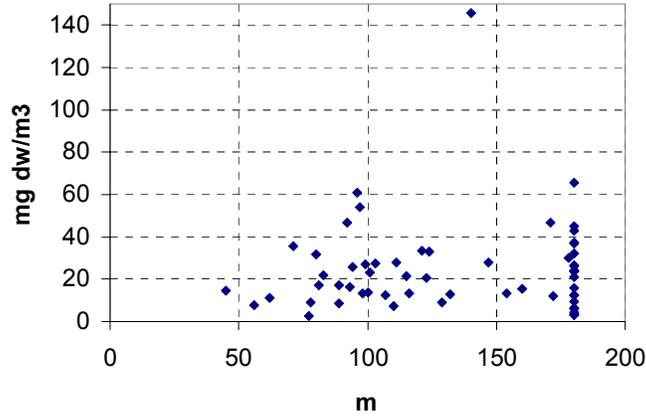**Figure 3-5. Concentrations of *C. finmarchicus* as a function of depths in top 50 m, cruise 9628, 3-14.April 1996. All depths greater than 180m are set to 180m.**

Therefore these conditions with the same *C. finmarchicus* concentrations on and off shelf are also tried in the model, see section 4.3.

3.3 *C. finmarchicus* mortality

In earlier versions of the model a constant mortality of *C. finmarchicus* has been applied. But since predation of *C. finmarchicus* is included in the mortality rate, we have tried to simulate this. The predation on *C. finmarchicus* is low in the spring but increases during May because spawning increases the amount of fish larvae, predating on *C. finmarchicus*. The *C. finmarchicus* mortality is computed as:

$$M_Z = m_Z(t) \cdot Z$$

$$m_Z(t) = \begin{cases} m_{Z,low} & JD \leq 120 \quad (before \quad 30.April) \\ \dfrac{(m_{Z,high} - m_{Z,low})}{140 - 120} \cdot JD + m_{Z,low} - \dfrac{(m_{Z,high} - m_{Z,low}) \cdot 120}{140 - 120} & 120 < JD < 140 \\ m_{Z,high} & JD \geq 140 \quad (After \quad 20.May) \end{cases}$$

$$m_{Z,low} = 0.01 - 0.05d^{-1} \quad m_{Z,high} = 0.15d^{-1}$$

Equation 3-6

4 Results from the modelling

All parameters used in the modelling are listed in Table 6-2, section 6. Equations can be found in section 6.1.

4.1 Horizontal exchange in the model

Looking closer at how volume is exchanged in the cylinder model, (Figure 4-1 and Figure 4-2) we see that with the average horizontal exchange coefficient of k_{typ}=1.45 km/d (Figure 4-1) it takes 2.5 months for the concentration in the innermost ring to reach half the concentration off shelf. Even if the advection is increased to 3 * k_{typ}=4.35 km/d it still takes almost a month for the concentration in the innermost ring to reach half of the

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concentration off shelf. These results fit with the reality, even though we sometimes see a very fast increase in the *C. finmarchicus* concentration on shelf after the individuals ascend from the deep water off shelf (Gaard and Hansen 2000).
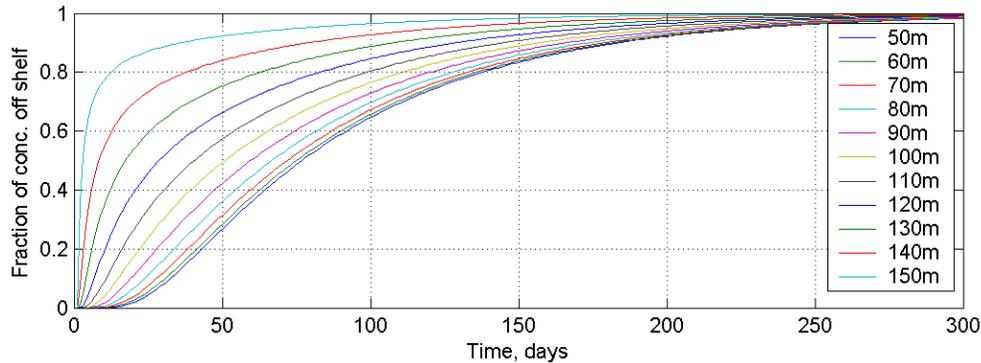
Figure 4-1. Advection onto the shelf with a constant concentration off shelf being 1 and a start concentration on shelf being 0. The horizontal exchange is k = 1.45 km/d = k_{typ}.
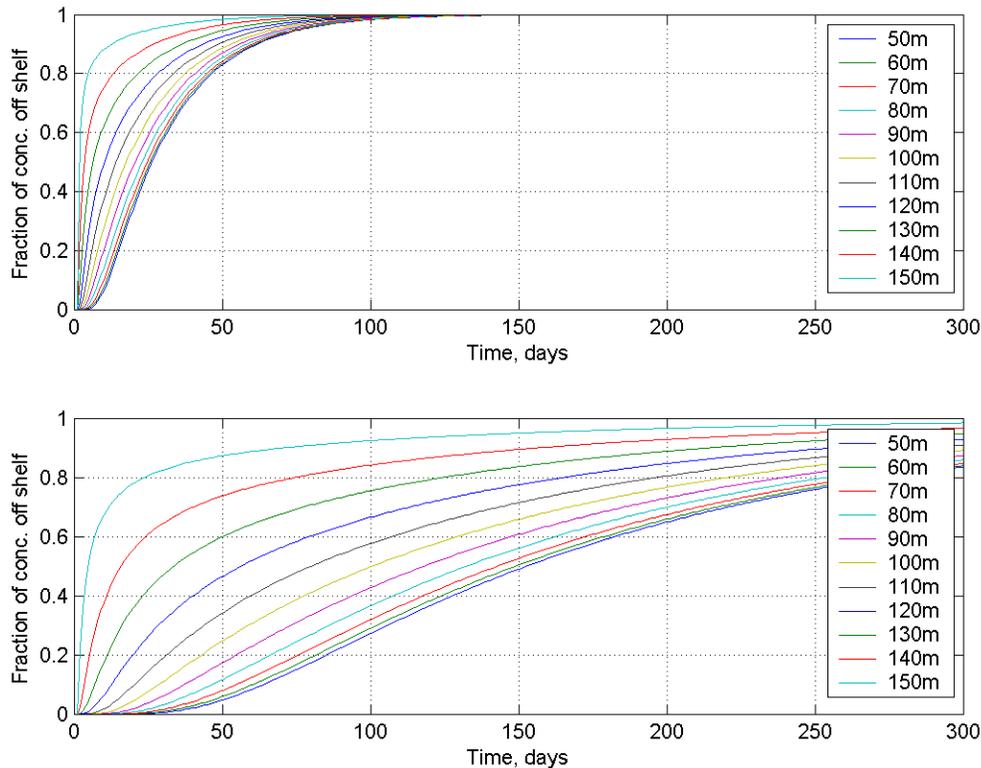
Figure 4-2. Advection onto the shelf with a constant concentration off shelf being 1 and a start concentration on shelf being 0. Upper panel: k = 4.35 km/d = $3*k_{typ}$. Lower panel: k = 0.725 km/d = $0.5*k_{typ}$.

4.2   Modelling results

Usually the model is initiated before the bloom, 1.April and runs until 1.June-1.July. The model has been run with different horizontal exchange coefficients (constant during one run) in order to see how the horizontal exchange influences the growth in the innermost

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ring, which corresponds to a station located centrally on the shelf. In all model runs below, Satel irradiance averaged over 1996-2000 has been used (www.satel-light.com). Figure 4-3 shows a model run with a low and a high exchange rate compared with observations. The low exchange rate is $0.5*k_{typ}$ and the high exchange rate is $3*k_{typ}$. With a low exchange rate the bloom starts in late April and reaches a maximum 13.May. This large bloom is limited by *C. finmarchicus*, which first are advected into the innermost ring and after that have had a good grazing environment and have grown large in biomass. This large grazing pressure is probably unrealistic, because in reality *C. finmarchicus* would spawn at this time, and therefore the biomass would not increase as much as we see here, because the grazed phytoplankton would be used to reproduction.
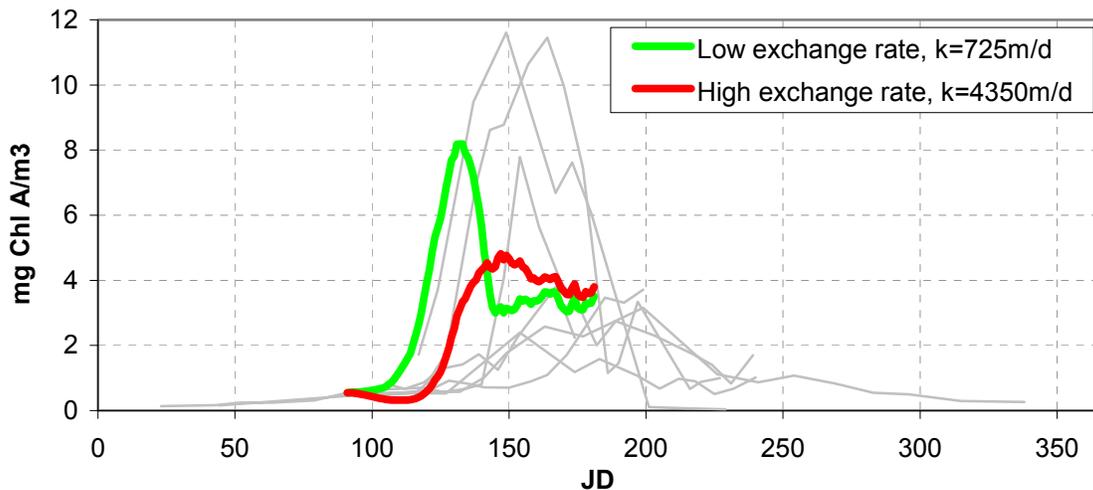
Figure 4-3. Modelled chlorophyll concentrations (thick colored lines) compared with observations from different years (thin gray lines).

With a high exchange rate we see that the bloom is low and delayed as expected (red line). It starts medium May and has a maximum in the end of May, but the question is whether this is due to a large import of *C. finmarchicus*, which grazes the phytoplankton or it is because of a continual export of the phytoplankton. Runs from the model, where we have set *C. finmarchicus* off shelf to zero, indicate that it is the latter, see Figure 4-4. This is a simple model and therefore the conclusion should be treated cautiously. But runs from the model indicate that the variable exchange of shelf water affects the timing of the start of the phytoplankton spring bloom more than grazing from *C. finmarchicus*.

Figure 4-4. Modelled chlorophyll concentrations with a high exchange rate (k=4350 m/d), with and without *C. finmarchicus* off shelf
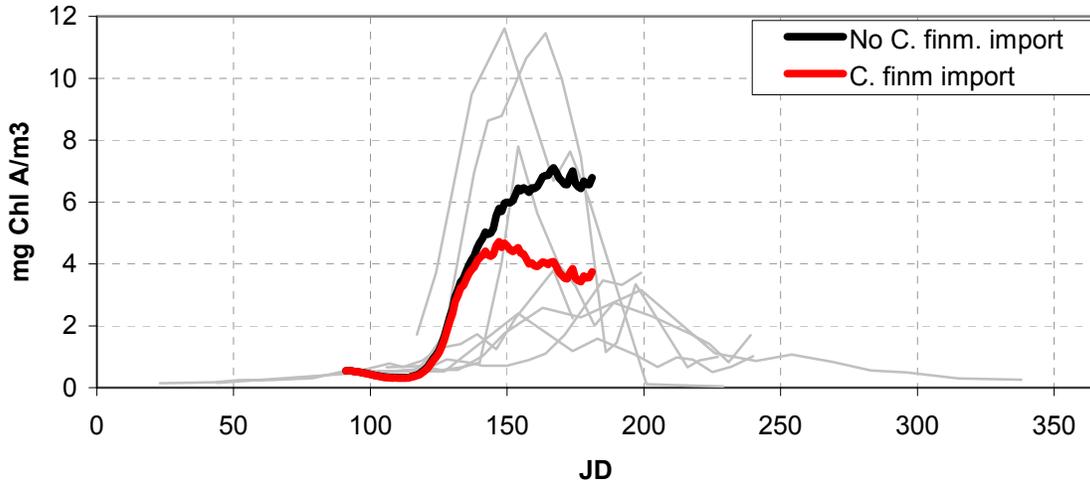
4.3   Runs with high start concentration of *C. finmarchicus* on shelf

As we see in Figure 3-5, some years *C. finmarchicus* are advected onto the shelf before the bloom starts. We wish to try this in the model and will assume a start concentration of *C. finmarchicus* on shelf of 40 mg dw/m^3, i.e. 16 mgC/m^3. This is an extremely high concentration of *C. finmarchicus*, corresponding to more than 100 adult C6 pr m^3, and it is probably very rare that such high concentrations of *C. finmarchicus* are found in the pre-bloom period. Note that some of the properties of *C. finmarchicus* have been changed in order to increase the survival of *C. finmarchicus*. This is shown in Table 4-1. The reason for this is that in the model it is very difficult for *C. finmarchicus* to survive on shelf in the start period due to the low phytoplankton concentrations, see Figure 4-5. In reality the animals probably can economize when there is a shortage of food, and this is partly implemented in the model by demanding that the respiration is a function of grazing:

$$R_Z = r_Z \cdot Z + r_{Z,A} \cdot a_Z \cdot G_Z \qquad\qquad r_Z = 0.1 d^{-1}$$

$$r_{Z,A} = 0.05$$

$$a_Z = 0.8 d^{-1}$$

Equation 4-1

where *G* is the amount grazed, *Z* is biomass, r_Z is the constant basic respiration and $r_{Z,A}$ is the activity respiration, but it seems as if this is not sufficient.

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	In Figure 4-3 and Figure 4-4	New constants
Basic respiration r_Z	0.1 d^{-1}	0.01 d^{-1}
C. finmarchicus mortality in April, $m_{Z,low}$, (increases during May to 0.15 d^{-1} in both cases)	0.05 d^{-1}	0.01 d^{-1}

Table 4-1. Properties of *C. finmarchicus*. In the text these rates are referred to as high and low loss rates respectively.
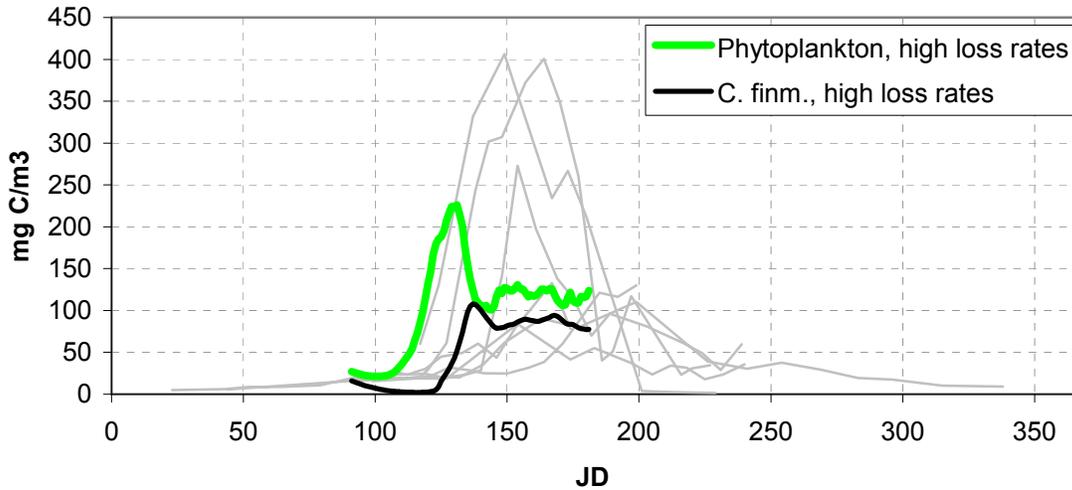

Figure 4-5. *C. finmarchicus* (black line) on shelf when it is assumed there is a high start concentration of *C. finmarchicus* and high loss rates (compare with Figure 4-3, Figure 4-4 and Table 6-2).

If we look at what is necessary for the *C. finmarchicus* population to suppress the phytoplankton bloom until May, we can look at how the phytoplankton is growing during April and May when only benthos grazes and compare it with when we have a high *C. finmarchicus* population on shelf already in April (see Figure 4-6). If we assume an initial concentration of *C. finmarchicus* consisting of 16 mgC/m^3 a phytoplankton concentration consisting of 0.5 mgChlA/m^3 and that the increase in phytoplankton is 0.05d^{-1}, i.e. 0.025 mg ChlA/m^3/d, the amount grazed by *C. finmarchicus* is 0.03 mg ChlA/m^3/d, which is comparable with the daily production of phytoplankton. This can also be seen in Figure 4-6, where we have plotted the phytoplankton behavior from two model runs, the red line has no *C. finmarchicus* at all in the system and the blue line has a high start concentration on shelf and advection from off shelf. From the figure we see that it should be possible for *C. finmarchicus* to suppress the bloom, provided that the growth rate of phytoplankton does not increase to much.
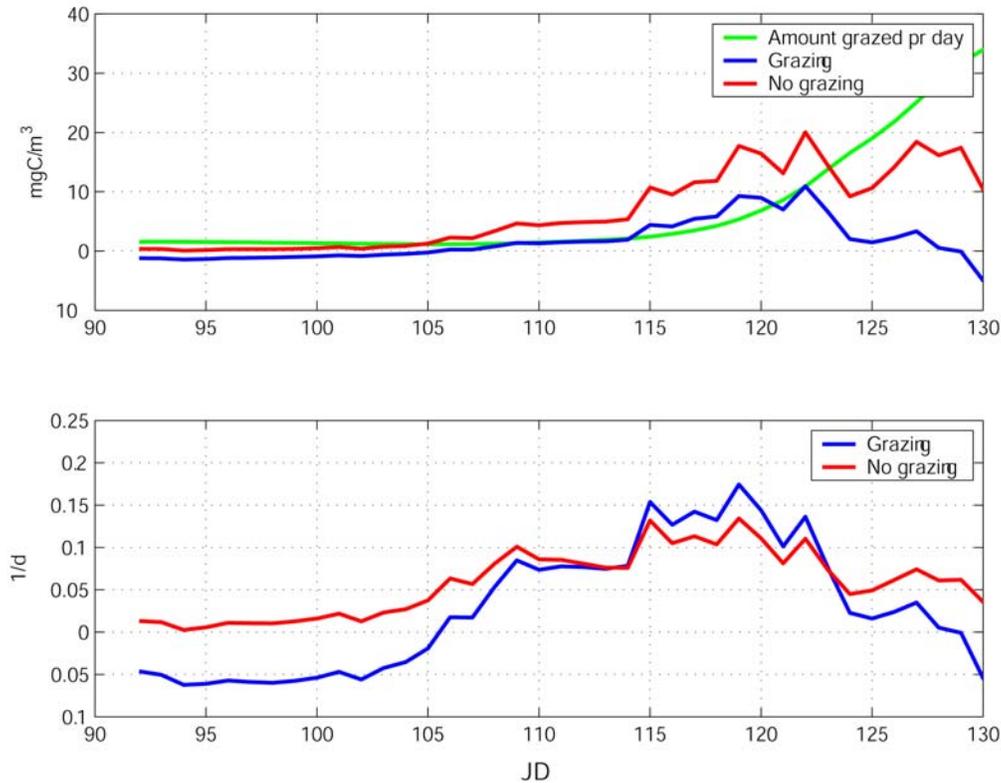
Figure 4-6. Daily increase in phytoplankton with and without *C. finmarchicus* grazing during April with low exchange. Upper panel: phytoplankton biomass change and amount grazed by *C. finmarchicus*. Lower panel: Net growth rate of phytoplankton. Average irradiance 1996-2000 is used and variations in irradiance are the only contributor to the variations seen, e.g. around day 120.

If we use the constants from Table 4-1 in a similar run as Figure 4-3, we see that the start period of the bloom is similar to Figure 4-3, while the maximum of the bloom is lower, see Figure 4-7, green and yellow line. High exchange of water delays the beginning of the spring bloom two weeks and the maximum is delayed 2.5 weeks.

If we look at the two runs with low exchange rate (pink and green line), we see that a high start concentration of *C. finmarchicus* delays the start of the bloom only one week and the maximums appears simultaneously.

The two runs with high exchange of water are quite similar regardless of the start concentration of *C. finmarchicus*, delaying the spring bloom 2-3 weeks compared with the early start of the spring bloom.
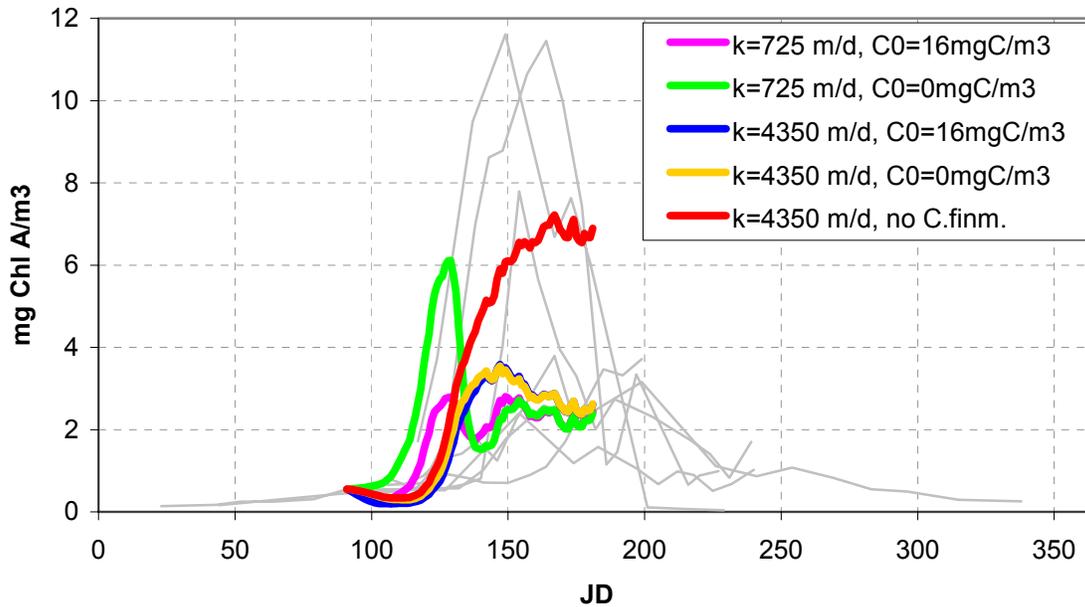
Figure 4-7. Model runs with low loss rates of *C. finmarchicus* and different exchange rates, start concentration of *C. finmarchicus* on shelf = 0 mgC/m^3 and 16 mgC/m^3. (This figure should be compared with Figure 4-3)

5   Conclusion

We see that it is large horizontal exchange, which delays the start of the spring bloom most and when the exchange rate is large there is almost no difference between a run with a high start concentration of *C. finmarchicus* and no *C. finmarchicus* in the beginning. It seems as if the amount of *C. finmarchicus* imported from off shelf has influence on the top of the bloom.

6 Appendix

Ring no	Max depth [m]	Mean depth [m]	Radius [km]	Surface area of rings [km²]	Volume of rings [km³]	Side area (exchange area) [km²]
0	(Land) 0	0	21.96	1514.3	0	
1	50	35. 1	26.04	615. 7	21.59	0
2	60	55.9	28.72	461.3	25.79	8.18
3	70	65.2	33.12	854.9	55.78	10.83
4	80	75.2	37.71	1021.3	76.80	14.57
5	90	84.7	41.67	988.7	83.73	18.96
6	100	95.8	46.22	1254.5	120.23	23.57
7	110	105.0	51.76	1705.5	179.02	29.04
8	120	114.8	55.75	1348.0	154.69	35.78
9	130	125.1	59.22	1252.8	156.75	42.03
10	140	134.8	62.41	1218.5	164.25	48.37
11	150	145.0	65.10	1079.6	156.52	54.90
12*		571.4		62114	35489	61.36

Table 6-1. Specifications of cylinder bottom model.

*This is off shelf area - and not a ring. Mean depth, surface area and volume have been computed.

α_{ext}	Gradient in attenuation coefficient equation. Named 'alfa_ext' in program code	0.0188	$\dfrac{m^3}{mgChlA \cdot m}$
a_B	Benthos assimilation. Named 'ab' in program code.	0.8	1/d
a_Z	Zooplankton assimilation. Named 'az' in program code.	0.8	1/d
d_{min}	Minimum parameter sedimentation rate (death rate of phytoplankton). Named 'd_min' in program code.	0.05	1/d
d_{max}	Maximum parameter sedimentation rate (death rate of phytoplankton). Named 'd_max' in program code.	0.5	1/d
D_0	Start detritus concentration on shelf. Named 'D' in program code.	0	$\dfrac{\mu molN}{kg}$
dt	Time step	1/48	d
D_u	Detritus concentration in the topmost 150m off shelf. Named 'Du' in program code.	0	$\dfrac{\mu molN}{kg}$
φ	Latitude in degrees. Is used to compute the sun angle above the horizon in order to compute the transmission of irradiance into the water. Named 'Lat' in program code	62	
F_0	Start flagellate concentration on shelf. Is in the pre-bloom period defined as 33/34 of the observed phytoplankton concentration the same day in 1997 (Debes, pers. comm.) Named 'F' in program code.		$\dfrac{mgChlA}{m^3}$
F_u	Flagellate concentration in the topmost 150m off shelf. Named 'Fu' in program code.	0	$\dfrac{mgChlA}{m^3}$
g_B	Maximum filtration rate for Benthos. Used while $P_{low}<P<P_{high}$. Named 'G_benthos' in program code.	0.0001	$\dfrac{m^3}{mgC \cdot d}$
g_Z	Maximum filtration rate for *C. finmarchicus*. Used while $P_{low}<P<P_{high}$. Named 'G_Calanus' in program code.	0.0036	$\dfrac{m^3}{mgC \cdot d}$
$\kappa_{I,F}$	Light half saturation constant for flagellates. Named 'Kappa_F' in program code	10e-6	E/m^2/s
$\kappa_{I,P}$	Light half saturation constant for diatoms.	110e-6	E/m^2/s

	Named 'Kappa_I' in program code		
κ_N	Half saturation constant for phytoplankton growth on nutrients and on death rate because of nutrient depletion. Named 'Kappa_N' in program code.	1	μmol N/kg
k	Horizontal exchange coefficient. Named 'diffusioncoefficient' in program code	725-4350	m/d
k_0	attenuation coefficient when there is no phytoplankton. Named 'k0' in program code	0.0644	1/m
mgChl_mgC	Conversion factor between mg ChlA → mgC for diatoms	35	$\dfrac{mgC}{mgChlA}$
mgChl_mgC_F	Conversion factor between mg ChlA → mgC for flagellates	50	$\dfrac{mgC}{mgChlA}$
mgC_mymol	conversion factor between mg C/m^3 → μmolN/kg	$\dfrac{16}{106 \cdot 12}$	$\dfrac{\mu molN \cdot m^3}{kg \cdot mgC}$
mgN_mymol	conversion factor between mg N/m^3 → μmolN/kg	1/14.35	$\dfrac{\mu molN \cdot m^3}{kg \cdot mgN}$
mgZ_mymol	conversion factor between mg zooplankton dryweight/m^3 → μmolN/kg	$\dfrac{0.4 \cdot 16}{106 \cdot 12}$	$\dfrac{\mu molN \cdot m^3}{kg \cdot mgZ}$
m_B	Mortality of benthos. Named 'm_Z_0' in program code.	0.001	1/d
m_Z	Mortality of *C. finmarchicus*. Is varying from 0.05d^{-1} in the spring to 0.15d^{-1} in the summer, see Equation 3-6. (Is also 0.01d^{-1} in pre bloom). Named 'm_Z_inc' in program code.		1/d
N_0	Start nutrient concentration on shelf. Named 'N' in program code.	12	$\dfrac{\mu molN}{kg}$
N_u	Nutrient concentration in the topmost 150m off shelf. Is a function Nu=Nu(t), see Figure 3-4 and Equation 3-5. Named 'Nu' in program code.		$\dfrac{\mu molN}{kg}$
P_{high}	Maximum phytoplankton concentration for maximum filtering. Named 'P_high' in program code.	5	$\dfrac{mgChlA}{m^3}$
P_{low}	Minimum phytoplankton concentration for zooplankton filtration. Named 'P_low' in program code.	0.2	$\dfrac{mgChlA}{m^3}$
$P_{max,F}$	Maximum photosynthesis for flagellates.	0.5	1/d

	Named 'P_maxF' in program code		
$P_{max,P.}$	Maximum photosynthesis for diatoms. Named 'P_max' in program code	3	1/d
P_0	Start diatom concentration on shelf. Is in the pre-bloom period defined as 1/34 of the observed phytoplankton concentration the same day in 1997 (Debes, pers. comm.) Named 'P' in program code.		$\dfrac{mgChlA}{m^3}$
P_u	Diatom concentration in the topmost 150m off shelf. Named 'Pu' in program code.	0	$\dfrac{mgChlA}{m^3}$
r_F	Flagellate basic respiration. Is given as $r_F + r_{F,A}$ *Flagellate assimilation. Named 'My_F' in program code	0.1	1/d
$r_{F,A}$	Flagellate activity respiration. Named 'My_F_a' in program code	0.1	
r_P	Diatom basic respiration. Named 'My_P' in program code	0.15	1/d
$r_{P,A}$	Diatom activity respiration. Named 'My_P_a' in program code	0.15	
r_B	Benthos basic respiration. Named 'My_z0' in program code.	0.001	1/d
$r_{B,A}$	Benthos activity respiration. Named 'My_z0_a' in program code.	0.005	
r_Z	C. finmarchicus basic respiration. Named 'My_z' in program code.	(0.01-) 0.1	1/d
$r_{Z,A}$	C. finmarchicus activity respiration. Named 'My_z_a' in program code.	0.05	
$T_{\frac{1}{2}}$	Half-life period for detritus. Named 'T2' in program code.	60	d
$Z_{0,inc}$	Start concentration of C. finmarchicus. Named 'Z0_inc' in program code.	0 (-16)	$\dfrac{mgC}{m^3}$
$Z_{0,perm}$	Start concentration of benthos. Is 0 for d>100m and 25mgC/m3 for d<=100m. Named 'Z0_perm' in program code.		$\dfrac{mgC}{m^3}$
Z_u	C. finmarchicus concentration in the topmost 150m off shelf. Is a function Zu=Zu(t), see Figure 3-3 and Equation 3-4. Named 'Zu' in program code.		$\dfrac{mgC}{m^3}$

Table 6-2. Model parameters.

6.1 Formulas

Formulas used in the ecosystem model are given below. The phytoplankton is divided into two distinct groups and zooplankton is also in two groups where both are assumed to be homogeneous biomasses, without spawning and characterized with growth only by grazing. The background for the equations can be found in Technical Report 04-02 (Eliasen, 2004).

Photosynthesis in the whole water column:

$$\langle photosynthesis \rangle = 1 + \frac{1}{kD} \ln \left(\frac{I_0 + \kappa_I e^{-kD}}{I_0 + \kappa_i} \right); \qquad D<0$$

$$k = 0.0644 \tfrac{1}{m} + \alpha_{ext} \cdot P \left[\frac{mgChlA}{m^3} \right]; \quad \alpha_{ext} = 0.0188 \tfrac{m^3}{mgChlA \cdot m}$$

for diatoms: $\kappa_{I,P} = 110 \tfrac{\mu E}{m^2 s}$, for flagellates $\kappa_{I,F} = 10 \tfrac{\mu E}{m^2 s}$

Phytoplankton gross growth:

$$G_P = P_{\max} \cdot \min(\langle photosynthesis \rangle, \frac{N}{\kappa_N + N}) \cdot P$$

(Index *P* or *F* should be added for diatoms and flagellates respectively)

$$\kappa_N = 1 \tfrac{\mu molN}{kg}, \quad \text{for diatoms: } P_{\max,P} = 3 \tfrac{1}{d}, \text{ for flagellates } P_{\max,F} = 0.5 \tfrac{1}{d}$$

Phytoplankton respiration:

Diatoms: $R_P = r_P \cdot P + r_{P,A} \cdot G_P$,

Flagellates: $R_F = r_F \cdot P + r_{F,A} \cdot G_F$

for diatoms: $r_P = 0.15 \tfrac{1}{d}$ and $r_{P,A} = 0.15$

for flagellates $r_F = 0.1 \tfrac{1}{d}$ and $r_{F,A} = 0.1$

Phytoplankton mortality. It is the same for both diatoms and flagellates:

$$M_P = \left(d_{\min} + \left(d_{\max} - d_{\min} \right) \cdot \exp\left(-\tfrac{N}{\kappa_N} \right) \right) P$$

$$M_F = \left(d_{\min} + \left(d_{\max} - d_{\min} \right) \cdot \exp\left(-\tfrac{N}{\kappa_N} \right) \right) F$$

$$d_{\min} = 0.05 d^{-1}, d_{\max} = 0.5 d^{-1}, \kappa_N = 1 \tfrac{\mu molN}{kg}$$

Grazing:

C. finmarchicus:

$$G_Z = \begin{cases} 0 & \text{if} \quad 0\frac{mgChlA}{m^3} \leq P+F < P_{low} \\ g_Z \cdot Z \cdot P & \text{if} \quad P_{low} \leq P+F < P_{high} \\ P_{high} \cdot ChlA:C \cdot g_Z \cdot Z & \text{if} \quad P_{high} \leq P+F \end{cases}$$

$$g_Z = 0.0036 \frac{m^3}{mgC \cdot d}$$

Benthos:

$$G_B = \begin{cases} 0 & \text{if} \quad 0\frac{mgChlA}{m^3} \leq P+F < P_{low} \\ g_B \cdot B \cdot P & \text{if} \quad P_{low} \leq P+F < P_{high} \\ P_{high} \cdot ChlA:C \cdot g_B \cdot B & \text{if} \quad P_{high} \leq P \end{cases}$$

$$g_B = 0.0001 \frac{m^3}{mgC \cdot d}$$

$ChlA:C = 1:35$ for diatoms and $ChlA:C = 1:50$ for flagellates

$P_{high} = 5$ mgChlA/m^3 , $P_{low} = 0.2$ mgChlA/m^3

C. finmarchicus respiration:

$$R_Z = r_Z \cdot Z + r_{Z,A} \cdot a_Z \cdot G_Z \qquad\qquad r_Z = 0.01d^{-1}(-0.1d^{-1})$$

$$r_{Z,A} = 0.05$$

$$a_Z = 0.8d^{-1}$$

Benthos respiration:

$$R_B = r_B \cdot B + r_{B,A} \cdot a_B \cdot G_B \qquad\qquad r_B = 0.01d^{-1}$$

$$r_{B,A} = 0.005$$

$$a_B = 0.8d^{-1}$$

C. finmarchicus mortality:

$$M_Z = m_Z(t) \cdot Z$$

$$m_Z(t) = \begin{cases} m_{Z,low} & JD \leq 120 \quad (before \quad 30.April) \\ \dfrac{(m_{Z,high} - m_{Z,low})}{140-120} \cdot JD + m_{Z,low} - \dfrac{(m_{Z,high} - m_{Z,low}) \cdot 120}{140-120} & 120 < JD < 140 \\ m_{Z,high} & JD \geq 140 \quad (After \quad 20.May) \end{cases}$$

$m_{Z,low} = 0.01 - 0.05d^{-1} \quad m_{Z,high} = 0.15d^{-1}$

Benthos mortality:

$$M_B = m_B \cdot B \qquad\qquad\qquad\qquad m_B = 0.05d^{-1}$$

Decomposed detritus material:

$$D_{Decomp} = \frac{\log 2}{T_{1/2}} D \qquad\qquad\qquad\qquad T_{1/2} = 60d$$

Differential equations:

Diatoms:
$$\frac{dP}{dt} = G_P - R_P - M_P - G_{Z,P} - G_{B,P}$$

Flagellates:
$$\frac{dF}{dt} = G_F - R_F - M_F - G_{Z,F} - G_{B,F}$$

Benthos:
$$\frac{dB}{dt} = a_B \cdot (G_{B,P} + G_{B,F}) - R_B - M_B$$

C. finmarchicus:
$$\frac{dZ}{dt} = a_Z \cdot (G_{Z,P} + G_{Z,F}) - R_Z - M_Z$$

Detritus:
$$\frac{dD}{dt} = M_Z + M_B + M_P + M_F - D_{decomp} + (1 - a_Z)(G_{Z,P} + G_{Z,F}) + (1 - a_B)(G_{B,P} + G_{B,F})$$

Nutrients:
$$\frac{dN}{dt} = R_Z + R_B + R_P + R_F + D_{decomp} - G_P - G_F$$

Horizontal exchange:
$$\frac{dC_i}{dt} = \frac{kA_{i+1/i}(C_{i+1} - C_i) - kA_{i/i-1}(C_i - C_{i-1})}{V_i},$$

$i+1, i, i-1$ represents rings in the model, C is concentrention of D, N, F, P or Z, A is exchange area, V is volume and k is horizontal exchange coefficient.

Off shelf all parameters are zero except nutrients and C. finmarchichus, which have functions describing their behaviour as a function of time:

$$N_U = \begin{cases} 12 \frac{\mu molN}{kg} & 0 < JD \leq 134 \\ (0.0008 \cdot JD^2 - 0.3532 \cdot JD + 44.931)\frac{\mu molN}{kg} & 134 < JD \leq 307 \\ 12 \frac{\mu molN}{kg} & 307 < JD \leq 365 \end{cases}$$

$$C.finm._U = \begin{cases} 7 \frac{ind}{m^3} \cdot 0.75 & 0 < JD \leq 50 \\ (-0.347 \cdot JD^2 + 7.8223 \cdot JD - 297.4)\frac{ind}{m^3} \cdot 0.75 & 50 < JD \leq 120 \\ (-0.2924 \cdot JD^2 + 108.98 \cdot JD - 8757.1)\frac{ind}{m^3} \cdot 0.75 & 121 < JD \leq 155 \\ 7 \frac{ind}{m^3} \cdot 0.75 & 155 < JD \leq 365 \end{cases}$$

where JD is the Julian day number.

7 References

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